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Abstract: Forest growth and yield models are important planning and decision-support tools for sustainable forest management. The Nova Scotia Growth and Yield (NSGNY) model is an empirical model developed using over forty years of local research data. It is a stand-level, single-species model that is site specific and applicable to even-aged natural and managed stands for dominant softwood and hardwood species in the province. The Province's growth and yield program was initiated in the 1990s with the development of normal yield tables for softwood species based on site index. These were then integrated with data from silvicultural treatments, including pre-commercial thinning, commercial thinning, and tree planting, and expanded into the softwood NSGNY model. Site index values for hardwood species were also developed and integrated into a hardwood growth and yield model in the 1990s and 2000s. The purpose of this study was to update the core functions of the NSGNY model for softwood plantations using new research trials and associated new permanent sample plots (PSPs). A total of 271 research PSPs with 1,222 measurements were included in the analysis, with the primary leading plantation species of red, white, black, and Norway spruce, white and red pine, and balsam fir. Linear and non-linear least squares regression were used to refit the original NSGNY model functions and develop new functions, such as product-specific volume estimates. The individual refit model functions showed acceptable model performance and fit, as reflected by root mean square error and mean bias. The new 2022 NSGNY model showed improved performance in all but one stand variable compared to the 2008 NSGNY model when validated against PSP data. The net effect of all updated functions and their interactions within the model on NSGNY output was a small and variable reduction in stand-level basal area and volume predictions, though total and merchantable volumes do eventually catch up to and exceed the original NSGNY output and show higher ratios of merchantable and sawlog stems. Future growth and yield research in Nova Scotia will include individual tree modelling to better address uneven-aged management as well as climate change adaptation.

Keywords: Forestry, growth and yield, plantations, modelling, Triad management

1. Introduction

Forest growth and yield models are important planning and decision-support tools for sustainable forest management that have a long history of research and application (Weiskittel et al., 2011). They support key planning steps in forest management such as forecasting stand dynamics, the timing of stand tending and harvest prescriptions, product merchandizing, and wood supply/timber management. Moreover, in forest planning, stand-level growth and yield models are often nested in a landscape-level modelling framework that integrates other model types (e.g., optimization) and other forest values (e.g., biodiversity, forest carbon), as is the case in Nova Scotia (Nova Scotia Department of Natural Resources and Renewables [NSDNRR], 2016). Note that NSDNRR reflects the current Department name and is used throughout the report for the sake of consistency, though past references may have been under different Department names.

Different kinds of forest growth and yield models exist, which are typically categorized as empirical or ecosystem process models, or a hybrid of the two (Korzukhin et al, 1996; Searls et al., 2021). Empirical models rely heavily on collected data from the system of interest, such as continuously measured growth data and site data, and therefore tend to be deterministic and regionally specific in nature (Korzukhin et al, 1996). Ecosystem process models, conversely, rely on ecophysiological processes and relationships and are more often used in research settings to understand ecosystem behaviour, rather than in a planning/implementation setting (Korzukhin et al, 1996). The Nova Scotia Growth and Yield (NSGNY) model is an empirical model developed using over sixty years of plot data and silvicultural trials. It is a stand-level, single-species model that is site specific and applicable to even-aged natural and managed stands for dominant softwood and hardwood species in the province.

In Nova Scotia, growth and yield modelling has its origins with the establishment of inventory permanent sample plots (PSPs) in the 1960s. Data from the PSPs were subsequently used to develop normal yield tables for softwood species based on site index (NSDNRR, 1990). These normal yield tables then integrated some important silvicultural treatments, including pre-commercial thinning, commercial thinning, and tree planting, and were evolved into the softwood NSGNY model (NSDNRR, 1993). Site index values for hardwood species were also developed (NSDNRR, 1987), which subsequently led to the development of a first (NSDNRR, 1997) and updated (O'Keefe & McGrath, 2006) hardwood NSGNY model.

Since the development of the softwood NSGNY model, there have been new silvicultural trials and new data generated from remeasurements of existing PSPs, especially for softwood plantations. Additionally, a recent independent review of forest practices in Nova Scotia (Lahey, 2018) recommended implementing ecological forestry and Triad zoning. The Triad involves delineating functional zoning on the landbase into areas of biodiversity conservation with no harvesting or silvicultural activities, areas of high production with intensive management focused on fibre production, and a much broader ecological matrix zone with a balance of harvesting, biodiversity conservation, and management of other forest values (Seymour & Hunter, 1999; Lahey, 2018; Himes et al. 2022). With regards to the high production forestry (HPF) zone, it was necessary to conduct an in-depth analysis on softwood plantation growth and yield modelling and wood supply to target high levels of productivity and product quality at shorter rotations (30-50 yr) and thereby minimize the footprint of the HPF zone on the total landbase (NSDNRR, 2021).

The purpose of this study was to update the core functions of the NSGNY model for softwood plantations using new softwood plantation trials and new research PSP data that have been generated since the creation of the softwood NSGNY model. This was necessary to support on-going implementation of ecological forestry and Triad zone management, notably the HPF zone. This study also ensures that the value of long-term silvicultural research trials in Nova Scotia are fully realized and integrated into planning and decision-support tools like the softwood NSGNY model.

2. Methods

2.1. Study Area

Nova Scotia is situated in the Atlantic Maritime Ecozone and Acadian Forest Region (Fig. 1). The province has a forested landbase of approximately 4.2 million ha (i.e., 76.7% of the total landbase), which contains both Acadian and Maritime Boreal ecosite groups and nine ecoregions (Neily et al., 2017). The long-term softwood plantation trials that were assessed in this study and the basis for NSGNY model updates are distributed across the province, though primarily in the more productive Acadian ecosites and especially in the Nova Scotia Uplands ecoregion (Fig. 1). Nova Scotia's climate is humid and moderated by its maritime setting and the North Atlantic Gulf Stream, with mild winters and cool summers (Neily et al., 2017). It has a mean annual temperature of 6.6 ^{o}C and mean total annual precipitation of 1,396 mm, with high levels of variability and frequent extreme weather conditions including hurricanes (Taylor et al., 2020).

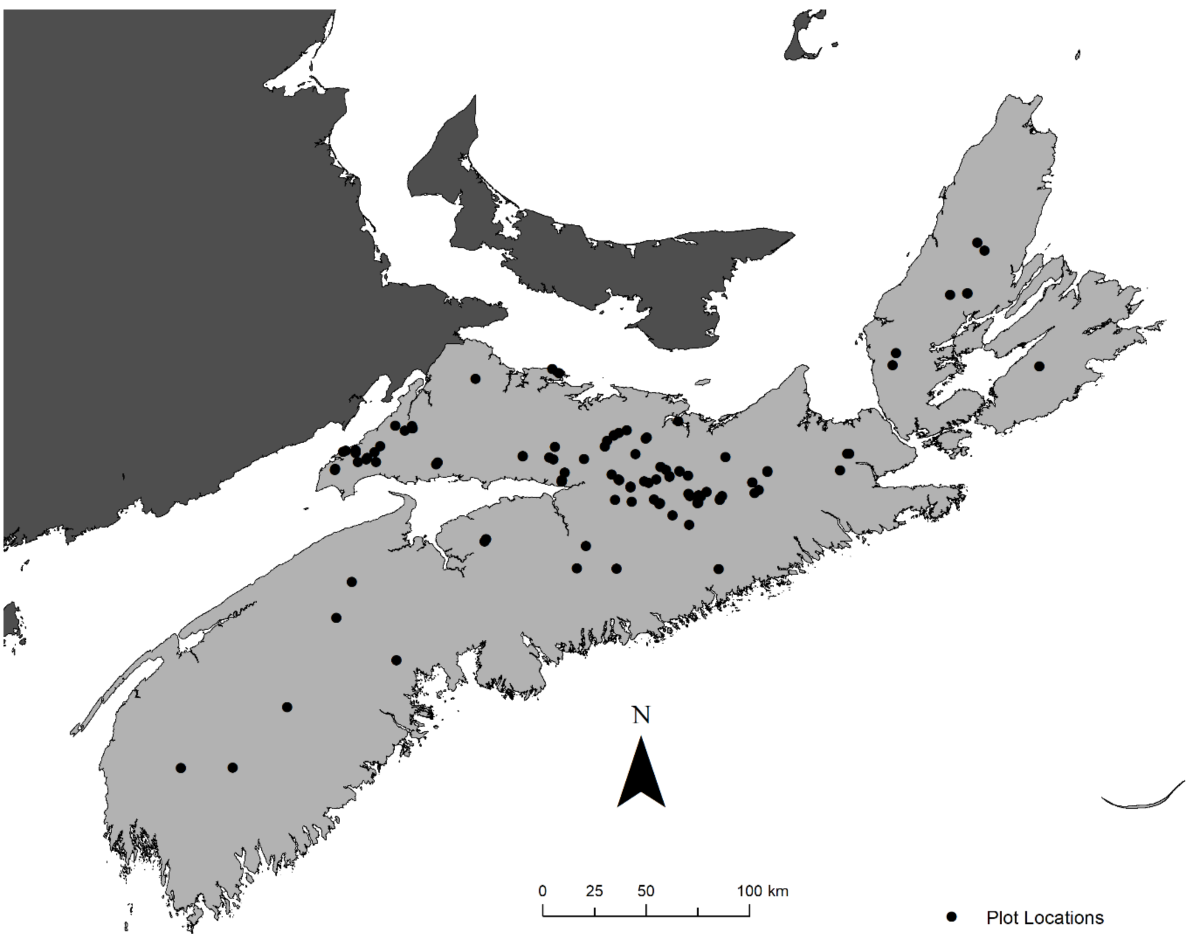


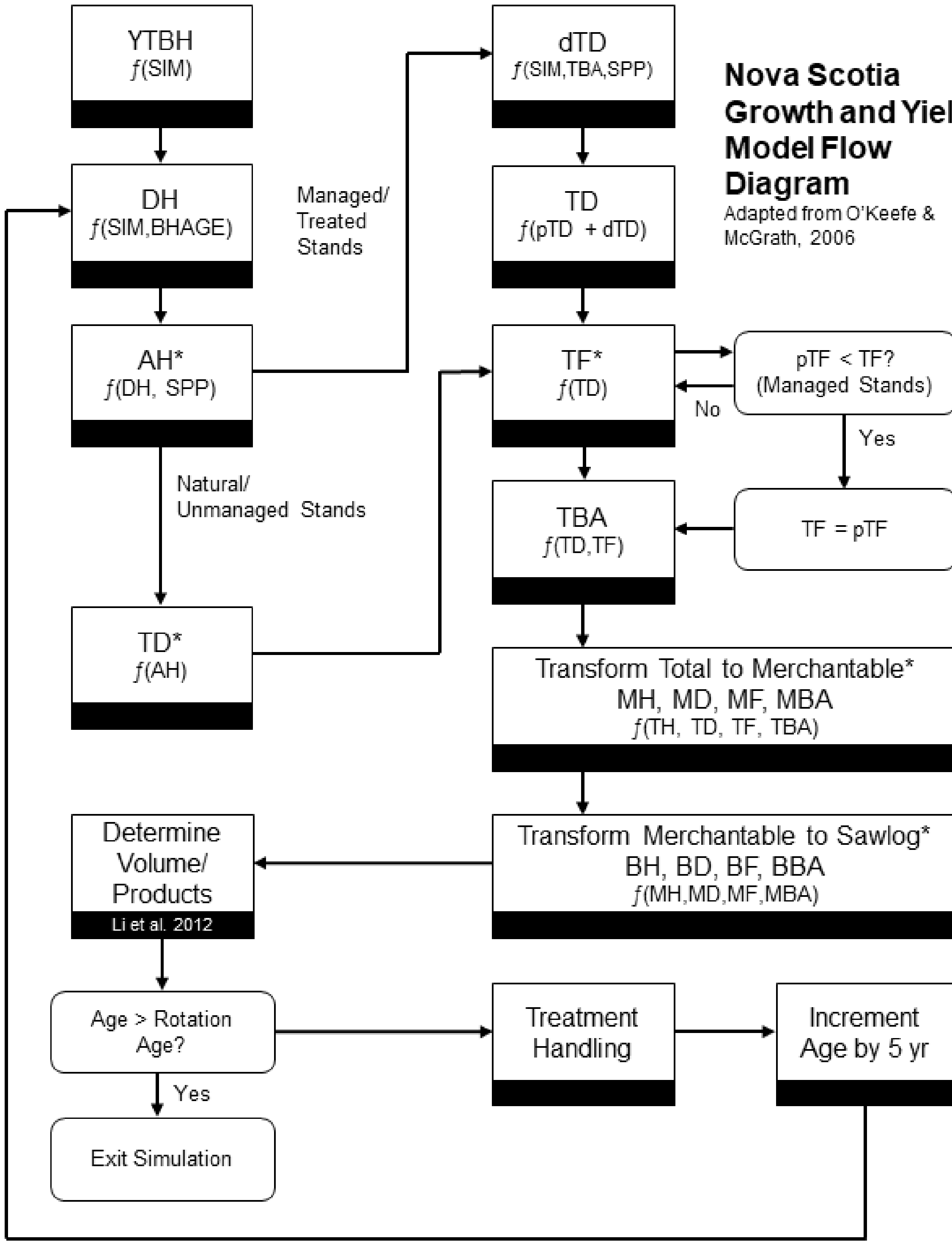
Fig. 1. Locations of research permanent sample plots in softwood plantations across Nova Scotia.

2.2. Model Description

The NSGNY model (Fig. 2; Table 1) is driven primarily by site index and species. It contains both softwood and hardwood models for natural stands and plantations, with treatment options for pre-commercial thinning and commercial thinning. The empirical, deterministic growth and yield model is based entirely on research PSP data collected in Nova Scotia over the span of four decades. It is a stand-level model that simulates an average tree in a stand, which is scaled to the stand level using user-defined stocking and calculated density (stems/ha) at five-year periods. Core model assumptions are that stands are even aged, fully stocked, dominated by a single species, site specific, and treatment specific (O'Keefe & McGrath, 2006), though stocking can be adjusted with a reduction factor. Note that land capability (LC), which is referred to throughout the report, is a direct translation of site index in volume-based units (i.e., m^3/ha/yr) and is a commonly used indicator of site productivity in Nova Scotia.

Model simulations are initiated by predicting years required for trees to reach breast height (1.3 m) as a function of site index and stump age, after which dominant and average heights are calculated at every period based on site index and breast height age. For natural stands, total diameter (i.e., quadratic mean diameter of all merchantable and unmerchantable trees in the stand) is a function of average height, therefore growth and yield over time are driven by site-specific height growth. For managed/treated stands, including the softwood plantations that are the focus of this report, a core model function predicts total diameter increment (i.e., the change in quadratic mean diameter between two periods) as a function of site index, total basal area, and species. When a treatment is scheduled, there are also functions predicting the artificial increase in total diameter and height as a result of commercial thinning or pre-commercial thinning from below.

Another core model function predicts total density as a function of total diameter, which is important in the scaling of stand-level variables such as basal area (m^2/ha) and merchantable volume (m^3/ha). Lastly, a series of ratio-based functions are used to calculate the ratio of merchantable and sawlog variables (i.e., height, diameter, density, and basal area) to total stand variables to derive the average merchantable and sawlog conditions in the stand. Merchantable volume is estimated using the species-specific taper equations for the Acadian region developed by Li et al. (2012), which replaced the Honer (1967) equations of earlier NSGNY versions and allows for product merchandising of tree-level PSP data to develop functions for predicting product ratios (see Section 3.7).



* Indicates separate functions for natural versus managed stands

Fig. 2. NSGNY model flow diagram as adapted from O'Keefe & McGrath, 2006. The *f*() denotation refers to a model equation that is the function of the variables included in brackets. Treatment handling refers to the model's implementation of a commercial

thinning treatment in the model for softwood plantations. All abbreviations used are defined in Table 1 below.

Table 1. Definition of key NSGNY model variables. All NSGNY diameter values are quadratic mean diameter, which is basal-area-weighted average diameter at breast height. Several NSGNY variables are categorized into total, merchantable, and sawlog categories based on DBH. Total refers to all trees within a stand while merchantable and sawlog refer to trees that meet the product specifications in the footnotes below and explained in detail in Tables 4 and 5.

Abbreviation	Description	Units	Updated in 2022
YTBH	Years to breast height	yr	Yes
DH	Dominant height	m	No
AH	Average height, which is basal-area-weighted average total height. In historical NSGNY documentation this was referred to as Lorey's height.	m	Yes
TD	Total diameter at breast height	cm	No
dTD	Five-year total diameter increment	cm	Yes
pTD	Total diameter at breast height in previous period	cm	N/A
TF	Total density. Density was historically referred to as frequency in previous NSGNY models.	stems/ha	Yes
pTF	Total density in previous period	stems/ha	N/A
TBA	Total basal area	m^2/ha	No
MH	Merchantable[1] average height	m	No
MD	Merchantable diameter at breast height	cm	No
MF	Merchantable density	stems/ha	Yes
MBA	Merchantable basal area	m^2/ha	Yes
BH	Sawlog[2] average height	m	No
BD	Sawlog diameter at breast height	cm	No
BF	Sawlog density	stems/ha	Yes
BBA	Sawlog basal area	m^2/ha	Yes
SIM	Site index, defined as dominant height at breast height age 50	m	N/A

[1] Merchantable is defined as having a DBH greater than 9 cm
[2] Sawlog is defined as having a DBH greater than 25.3 cm for hardwoods and 14.2 cm for softwoods. Sawlog was formerly referred to as board in historical NSGNY documentation.

2.3. Data Overview and Preparation

A total of 271 research PSPs with 1,222 measurements from 1978 to 2019 were included in the analysis (Table 2; Fig. 3 & 4), with the primary leading plantation species of red spruce (*Picea rubens*), white spruce (*P. glauca*), black spruce (*P. mariana*), Norway spruce (*P. abies*), white pine (*Pinus strobus*), red pine (*P. resinosa*), and balsam fir (*Abies balsamea*). The sample size was occasionally reduced in some of the statistical analyses for different NSGNY functions, which is described later in this section. Only research PSPs (i.e., not inventory PSPs) were used in this analysis.

Table 2. Study sample size, showing total number of permanent sample plots (and measurements) for each plantation species by land capability class. Species abbreviations include red spruce (RS), white spruce (WS), black spruce (BS), Norway spruce (NS), white pine (WP), red pine (RP), and balsam fir (BF).

Leading [1]	Land Capability (m^3/ha)											Total
	3	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	
RS			9(42)	13(61)	10(39)	3(13)						35(155)
WS		1(7)	2(9)	6(26)	14(76)	11(43)	5(22)	1(4)				40(187)
BS	3(16)	3(15)	12(52)	10(44)	11(47)	2(7)						41(181)
NS			3(12)	5(18)	7(34)	12(55)	8(32)	8(48)	4(32)	1(4)	2(9)	50(244)
WP			1(3)	1(2)	1(4)	2(6)	3(12)		1(6)			9(33)
RP			1(6)	2(10)	12(73)	16(85)	8(30)	11(42)	1(9)			51(255)
BF				1(3)	2(9)	4(14)	6(23)	1(4)				14((53)
Other [2]					5(18)	5(19)	6(13)	4(13)	4(22)	5(20)	2(9)	31(114)
Total	3(16)	4(22)	28(124)	38(164)	62(300)	55(242)	36(132)	25(111)	10(69)	6(24)	4(18)	271(1222)

[1] Leading refers to the leading species by basal area in a research permanent sample plot.
[2] Other species include tamarack (*Larix laricina*), European larch (*L. decidua*), European X Japanese larch (*L.* X *eurolepis*), Japanese larch (*L. kaempferi*), jack pine (*P. banksiana*), Scots pine (*P. sylvestris*), and Sitka spruce (*S. sitchensis*).

The softwood plantation trials have had varying numbers of research PSPs established in each trial, with PSPs then measured on a five-year cycle. Plot-level measurements include species, quadratic mean diameter (QMD) at breast height (cm), dominant height (m), average height (m), basal area (m^2/ha), density (stems/ha), stump age (yr), breast height age (yr), and site index at breast height age 50 (m). The variability and range of a selection of these variables are shown in Fig. 3 and 4. For a description of research PSPs and how they differ from inventory PSPs see NSDNRR (2004) and O'Keefe & McGrath (2006). Within the 271 research PSPs in softwood plantation trials, 23 plots (147 measurements) were commercially thinned at some point.

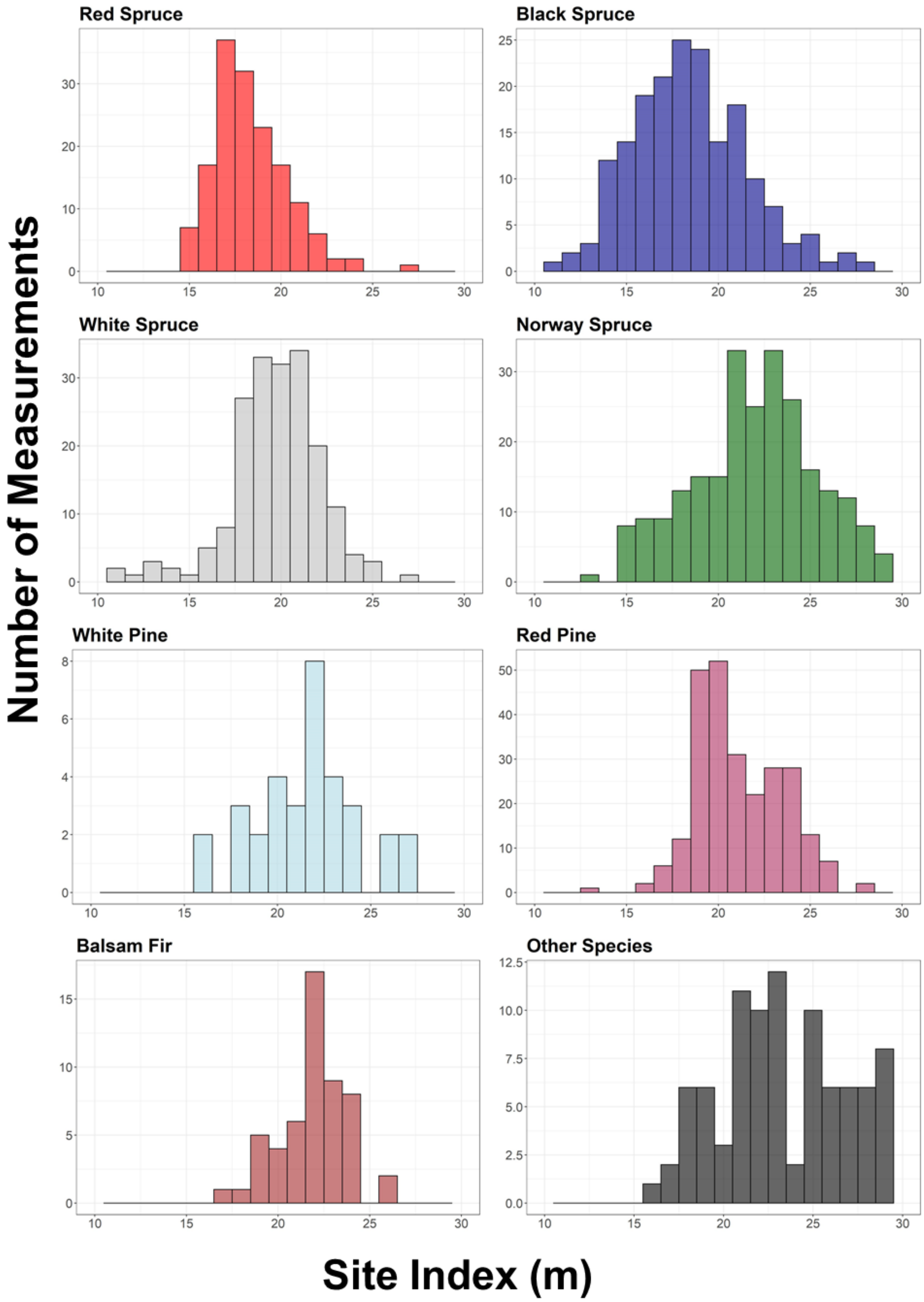


Fig. 3. Range and number of sample measurements from research PSPs across site index classes and grouped by species.

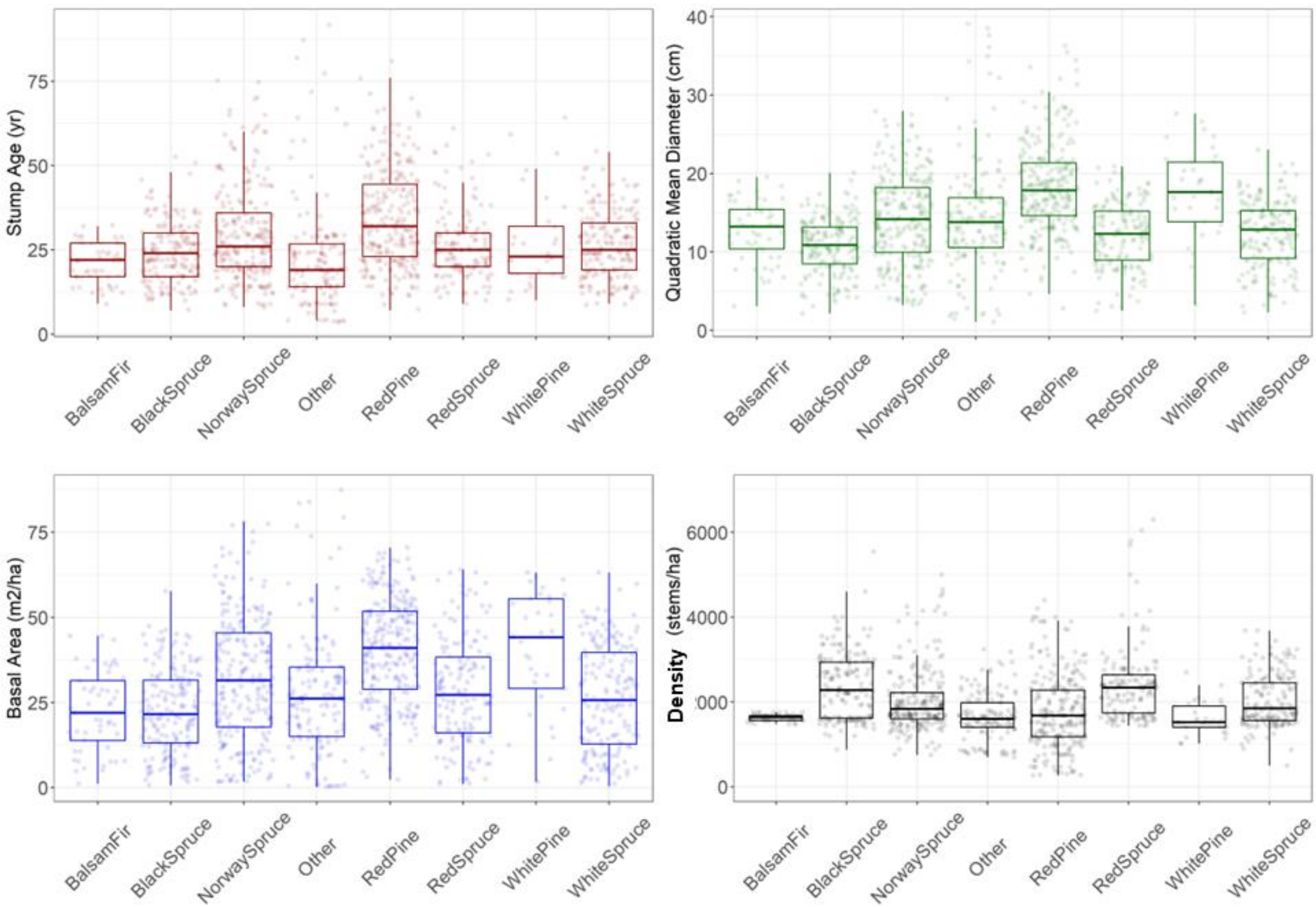


Fig. 4. Distribution of stump age, diameter, basal area, and density measurements grouped by species.

2.4. Data Analysis and Model Updates

Linear and non-linear least squares regression in R (R Core Team, 2022) were used to refit original NSGNY model functions (Table 3) with the full suite of 271 research PSP data from softwood plantations measured between 1978 to 2019, inclusive of the many re-measurements that have occurred since original model development in 1993 and the most recent 2008 NSGNY version. See Appendix B for the 2008 functions and coefficient values that were analyzed in this study. Note that several of the original equations and their coefficients require imperial units and subsequent conversion to metric units. Model fit was evaluated using root mean squared error (RMSE), mean bias (MB), and, when using linear regression, the coefficient of determination (R^2). For all PSPs, site index was corrected so that for a given plot the site index value was taken from the measurement closest to breast height age 50 then applied to all other measurements from that plot. The relationships between forest variables analyzed in this study are long-established in forest growth and yield theory, and the purpose of developing these regression models was for prediction rather than inference (e.g., hypothesis testing).

2.4.1. Years to Breast Height

For the years to breast height (YTBH) function, only those plantations where a cleaning (i.e., removal of competing unmerchantable and non-crop trees) had occurred were used to ensure height growth was as accurate a picture of the site as possible for the model's intended application in free-to-grow stands, yielding a sample size of 142 plots with 560 measurements. However, YTBH does not vary between measurements, so 142 was the final size of data set used for modelling. YTBH values were calculated as the difference between stump age and breast height age.

2.4.2. Average Height

Average height was refit and predicted using a linear model with quadratic terms. No softwood plantation plots were removed from the analysis of average height, though regression models were built with and without species terms (i.e., dummy variables) for the sake of comparison.

2.4.3. Total Density

The total density function predicts density as a function of total diameter and provides information on stand stocking and relative competition. Plantations present a challenge for developing this function compared to natural stands because of the control of competition through planting density and silviculture treatments. The underlying purpose of this function is to predict tree mortality from competition and stem exclusion. However, in plantations, trees are planted at spacings where competition with neighbouring trees does not occur in the early years of stand development. Consequently, to fit the regression model for this function we removed measurements that had an annual mortality rate of less than 1% in order to remove measurements in younger plantations where any competition-induced mortality had yet to occur. To capture the faster diameter growth of plantations with full stocking, we also only included plots that had a stand density at a given diameter greater than what would be predicted for natural stands using the NSGNY density function for natural stands. Note that this natural density function has also been updated since the 2008 NSGNY version. The updated coefficients can be found in Appendix A.

The final analysis included 25 plots with 65 measurements. While the total density function is non-linear, it was fit using linear regression with log-transformed variables. Models were fit with and without species terms.

2.4.4. Merchantable and Sawlog Conversion

Four ratio-based equations were also refit, all with the same non-linear function. These included the ratio of both merchantable density (MFrat) and basal area (MBArat) to total density and basal area, as well as the ratio of sawlog density (BFrat) and basal area (BBArat) to merchantable density and basal area. The merchantable density functions included all plots and measurements. The sawlog density ratio functions only included measurements with a merchantable diameter greater than 11.2 cm, which yielded a sample size of 269 plots and 963 measurements. The sawlog basal area ratio functions only included measurements with a merchantable diameter greater than 11.1 cm, yielding a sample size of 270 plots and 984 measurements. These merchantable diameter cut-off values were the intercept (i.e., a ratio of 0) from the original NSGNY functions.

2.4.5. Five-Year Diameter Increment

The final function to be refit with the full PSP dataset from softwood plantations was the five-year diameter increment function (a.k.a., delta total diameter [dTD]). Unlike the analysis and refitting of all previous NSGNY functions, the dTD regression analysis also involved an *a priori* change in the function type. We used a newer, more simplified dTD function derived from a recent study on commercial thinning in natural, pre-commercially thinned, and planted stands across Nova Scotia (McGrath, 2007). The function predicts dTD from site index (m) and total basal area (m^2/ha) only and does not require spacing as an input. Plots with excessively high annual mortality rates (i.e., greater than 2%) or negative mortality rates (i.e., ingrowth exceeds mortality) were excluded from the analysis to ensure the diameter increment was driven by actual growth and not excessive changes in density. This was done to remove anomalous growth records from the analysis, but the threshold for doing so is not based on any existing literature, and as such is a source of uncertainty and an area for future research. Also, because the dependent variable was diameter increment between two periods, the most recent measurement period of any given plot could not be included, giving a final sample size of 215 plots and 853 measurements. Species terms (and combinations thereof) that were tested included 1) black spruce and red pine, 2) white spruce and black spruce combined along with red pine, 3) black spruce and white spruce, and 4) black spruce, white spruce, and red pine combined with white pine. The best model fit led to the inclusion of white spruce and black spruce as species terms in the function, with the other species category including only Norway spruce, red spruce, and red pine.

2.4.6. Product Volume Estimates

Lastly, a number of new functions were created in the 2022 model update. Since NSGNY is an average tree model, the quadratic mean diameter (i.e., TD, MD, and BD) and average height (i.e., AH, MH, and BH) can be used to calculate volume of the average tree for each utilization class (Table 4) that then gets expanded by stem density (i.e., TF, MF, and BF) to produce stand-level volume estimates. Previous to this model update, all model volumes were based on Honer (1967) volume equations. This 2022 update based volume estimation on taper equations developed by Li et al.(2012). In addition to volume estimation, these taper equations allow for product merchandising of individual trees as a function of quadratic mean diameter. As a result, softwood plantation trees were merchandised for three general product classes: sawlog (first priority), studwood (second priority), and pulpwood (third priority) as defined in Table 5.

Table 4. Tree size classification used for PSP compilations.

Tree Size Class	Minimum DBH Outside Bark		Minimum Top Diameter Inside Bark		Stump Height	
	(cm)	(in)	(cm)	(in)	(m)	(ft)
Total	none	none	none	none	none	none
Merchantable	9.10	3.6	7.62	3.0	0.15	0.5
Sawlog	14.22	5.3	10.16	4.0	0.15	0.5

Table 5. Product classes used for softwood plantation PSP compilations.

Product Class	Minimum Top Diameter Inside Bark		Maximum Butt Diameter Inside Bark		Log Lengths	
	(cm)	(in)	(cm)	(in)	(m)	(ft)
Pulpwpood	7.62	3.0	none	none	2.4,3.0,3.6	8,10,12
Studwood	10.16	4.0	35.56	14.0	2.4,2.7,3.0	8,9,10
Sawlog	15.24	6.0	none	none	3.0,3.6,4.9	10,12,16
Waste	7.62	3.0	none	none	none	none

1 Multiple values indicate the different log lengths allowed for each product class.

2 Waste indicates sections of merchantable logs that are not of sufficient length to meet product specifications.

This product estimation used a ratio fitting approach (Burkhart, 1977) similar to the merchantable and sawlog conversion functions using non-linear regression with data only from red spruce, white spruce, and Norway spruce plantations. The approach used three ratio functions, with the first predicting the ratio of pulpwood and waste combined to merchantable volume with a sample size of 471, where waste is any leftover merchantable length that cannot be utilized. The second function predicted the ratio of just pulpwood to merchantable volume, again with a sample size of 471. The third function predicted the ratio of sawlog volume to all sawables (i.e., sawlog and studwood combined) and had a sample size of 272 because of the lower number of sawlog-sized trees in the sample data. With these three ratio functions, merchantable volume can be divided into associated product classes.

2.4.7. Post-Establishment Stocking Adjustment

Other new functions added to the 2022 model update were created to introduce post-establishment mortality in softwood plantations that was separate from density-dependent mortality. Plantation research PSPs were by design established in fully stocked portions of plantations. Care was taken in maintenance of plots for competition control and to keep them in a free-to-grow condition (i.e., of sufficient height to avoid competition from non-crop trees). However, past exploratory data analysis showed there was some mortality in planted stock prior to any mortality associated with competition (i.e., the density function), which was more pronounced in white spruce compared to red spruce. While this is an area that requires further research, to reflect these observations in the model, simple stocking-time-dependent relationships were developed for the main softwood plantation species. These are implemented as a periodic stocking update that does not impact any of the core model growth functions. A simple linear model was fitted for red spruce, white spruce, black spruce, Norway spruce, and red pine with stump ages between 20 and 50. The function predicted stocking reductions as the ratio of measured total basal area to predicted total basal area (i.e., predicted QMD and total density) as a function of stump age. Note that this function can be deactivated in the updated model interface if users wish to only apply the user-specified stocking factor to reflect local knowledge of stand and site conditions.

2.5. Model Comparison and Validation

We quantitatively compared the updated NSGNY model against the original 2008 model using RMSE as a percent of the sample mean and MB. We also validated both the 2008 and updated 2022 NSGNY models against all PSP data for softwood plantations (i.e., not just the data used to fit individual functions). The sample size for model validation against PSP data for total stand variables was 1512 measurements, excluding total diameter (TD) that had a sample size of 1190 (because TD is based on diameter increment where the first measurement cannot be included). The sample size for all merchantable stand variables was 1326, as some younger plot measurements had no merchantable trees and were therefore excluded.

3. Results

All model equations were successfully refit with general improvement in RMSE, MB, and R^2, with the exception of YTBH (Table 6). The following subsections present the results for each of the functions or function grouping.

Table 6. Updated coefficient values for the 2022 NSGNY model functions assessed in this study. Coefficients are rounded here but presented in full in Appendix A. All log functions are log base 10.

Function Code	Coefficients β_0	β_1	β_2	β_3	β_4
YTBH	44.070	-1.636	0.022	-0.0001	
AH	-0.534	0.943	0.037	0.040	
TF	5.570	-1.761			
dTD	0.185	1.010	-0.027	-0.601	-0.405
Mfrat	-0.514	77.395			
Bfrat	-0.458	497.017			
MBArat	-0.646	140.444			
BBArat	-0.576	1745.149			
PLP+WST	9.323	-0.233			
PLP	3.433	-0.187			
LOG	-0.288	159.614			

Functions:

$YTBH = \beta_0 + \beta_1 SI + \beta_2 SI^2 + \beta_3 SI^3$
Where YTBH is years to breast height (yr) and SI is site index at breast height age 50 (m)

$AH = \beta_0 + \beta_1 DH + \beta_2 RP + \beta_3 DHRP$
Where AH is average height (m), DH is dominant height (m), RP is the red pine species term (0/1), and DHRP is a red pine-dominant height interaction term

$TF = 10^{(\beta 0 + \beta 1 \log(TD))}$
Where TF is total density (stems/ha) and TD is total quadratic mean diameter (cm)

$Mfrat = (1 – e^{(\beta 0TD)})^{\beta_1}$
Where Mfrat is the ratio of merchantable density (stems/ha) to total density (stems/ha) and TD is total mean quadratic diameter (cm)

$Bfrat = (1 – e^{(\beta 0MD)})^{\beta_1}$
Where Bfrat is the ratio of sawlog density (stems/ha) to merchantable density (stems/ha) and MD is merchantable mean quadratic diameter (cm)

$MBArat = (1 – e^{(\beta 0TD)})^{\beta_1}$
Where MBArat is the ratio of merchantable basal area (m^2/ha) to total basal area (m^2/ha) and TD is total mean quadratic diameter (cm)

$BBArat = (1 – e^{(\beta 0TD)})^{\beta_1}$
Where BBArat is the ratio of sawlog basal area (m^2/ha) to merchantable basal area (m^2/ha) and MD is merchantable mean quadratic diameter (cm)

$dTD = (\beta_0 SI + \beta_1)\, e^{(\beta_2 TBA)} + \beta_3 BS + \beta_1 WS$
Where dTD is five-year diameter increment (cm), SI is site index (m), TBA is total basal area (m^2/ha), BS is the black spruce species term (0/1), and WS is the white spruce species term (0/1)

$PLP+WST = \beta_0 e^{(\beta 1TD)}$
Where PLP+WST is the ratio of pulpwood and waste volume combined (m^3/ha) to merchantable volume (m^3/ha) and TD is total mean quadratic diameter (cm)

$PLP = \beta_0 e^{(\beta 1TD)}$
Where PLP is the ratio of pulpwood volume combined (m^3/ha) to merchantable volume (m^3/ha) and TD is total mean quadratic diameter (cm)

$LOG = (1 - e^{(\beta 0TD)})^{\beta 1}$

Where LOG is the ratio of sawlog volume (m^3/ha) to studwood and sawlog volume combined (m^3/ha) and TD is total mean quadratic diameter (cm)

3.1. Years to Breast Height

The analysis of YTBH showed high levels of variability across species (Fig. 5) and sites (Fig. 6) with little evidence of a discernable trend. Moreover, the original predicted curve is more biologically defensible, with increasing years to breast height on poorer sites. Regression analysis to refit the original softwood YTBH function showed a near absence of a trend (Fig. 6), with an RMSE of 1.1 yr, MB of -1.7, and none of the coefficients being statistically different from zero. We determined that refitting the YTBH function for softwood plantations only was not meaningful and future research on YTBH should be conducted on the entire softwood NSGNY model including natural and managed/treated stands, with the expectation that more productive sites or faster growing species will have lower YTBH values.

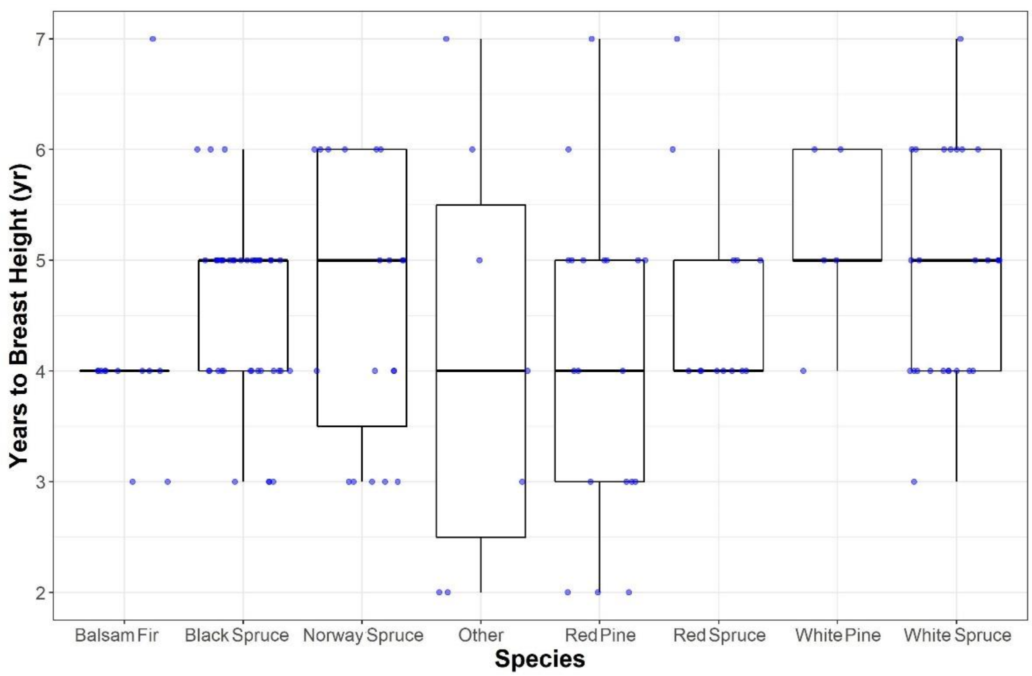


Fig. 5. Distribution of years to breast height observations grouped by species.

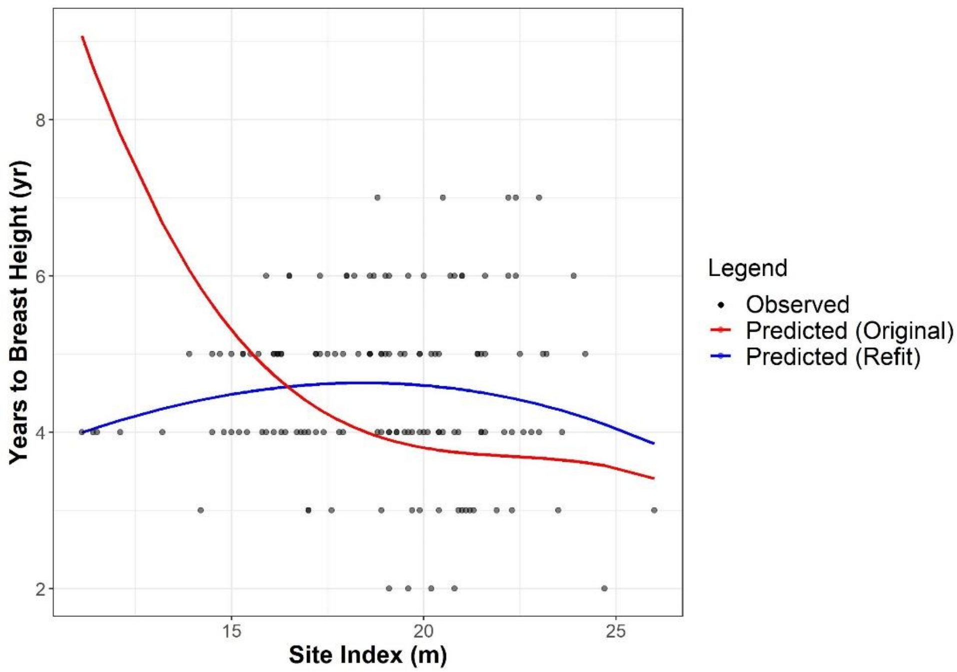


Fig. 6. Observed and predicted years to breast height values, showing both the original and refit function for softwood plantations.

3.2. Average Height

The analysis of average height under different model iterations (i.e., linear and non-linear, with and without species terms) led to two conclusions: 1) a linear model was better suited to the dataset and 2) including both a red pine species term (i.e., dummy variable) and interaction variable for red pine and dominant height gave the best fitting model. The average height model had RMSE value of 0.4 m, MB value of < -0.001, and an R^2 value of 99.4% (Fig. 7), indicating a close fit to observed data with a very small over prediction. The refit results closely match those of the original function for other species, while the red pine coefficient value shows higher average heights for that species.

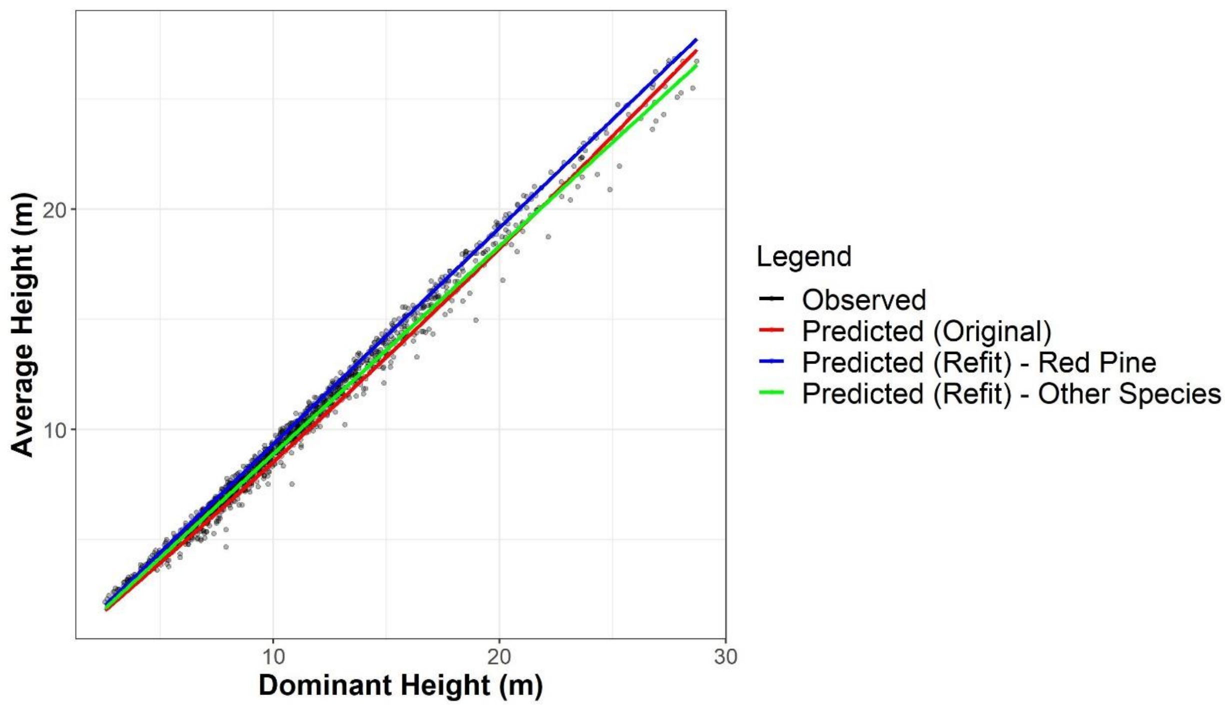


Fig. 7. Observed and predicted average height values, showing both the original and refit function.

3.3. Total Density

The final total density model (Fig. 8) did not include species terms, as these reduced model fit (i.e., RMSE, MB, and R^2) and were likely not appropriate given the smaller sample size used in this regression analysis compared to other analyses. The model had a RMSE of 172.9 stems/ha, MB of 7.8 stems/ha, and R^2 of 93.0%. Both the original and refit plantation functions predict a higher density at a given average stand diameter than the model for natural stands. However, the refit plantation function that includes updated PSP data since the original NSGNY function was developed is lower than the original. Recall that given the model flow control for managed stands, this function would not be called to calculate total density until it predicted a density that is lower than the previous period, which for the case of plantations is the total density at the time of planting (e.g., 2000 stems/ha).

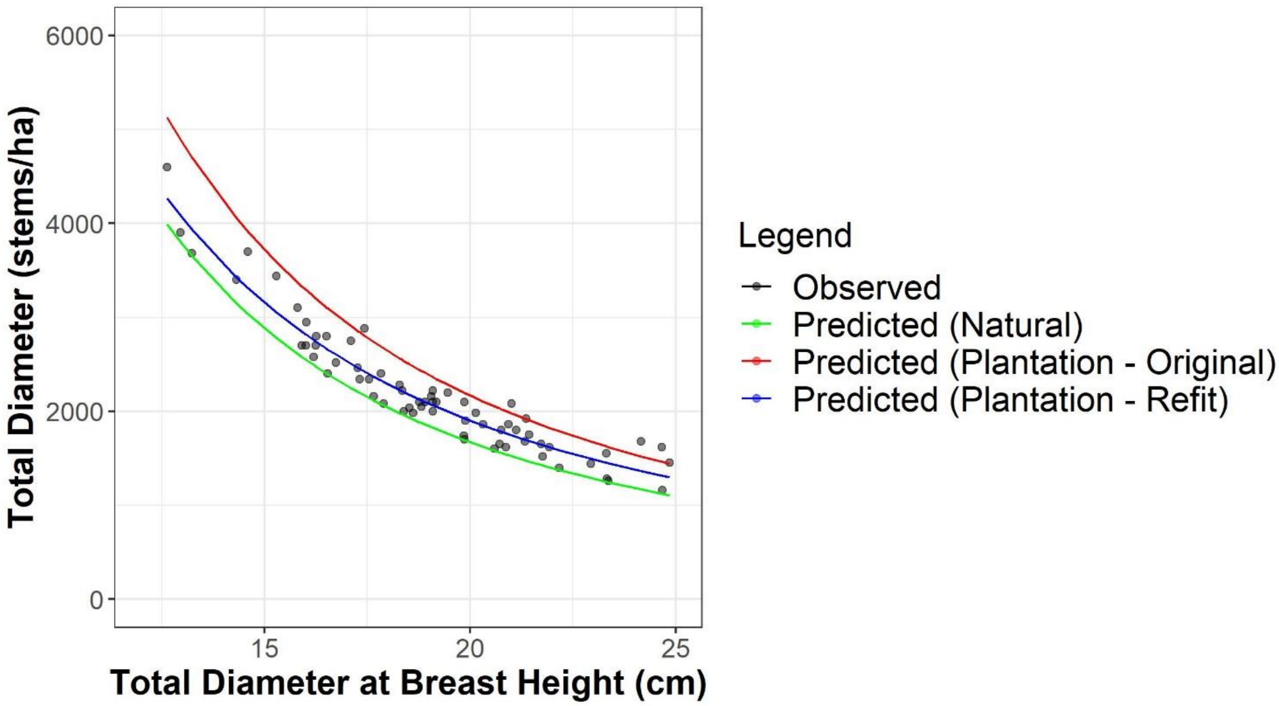


Fig. 8. Observed and predicted total density values, showing both the original and refit function for softwood plantations alongside the function for natural softwood stands.

3.4. Merchantable and Sawlog Conversion

The conversion ratios for merchantable and sawlog density and basal area (Fig. 9) all showed relatively good fit and considerably better alignment with PSP data from softwood plantations compared to the original NSGNY functions. The density ratio models had RMSE values of 0.06 and MB values of -0.007 for both merchantable and sawlog models, indicating a slight over prediction. Basal area ratio models had RMSE values of 0.05 for both models, while MB values were 0.0004 and -0.004 for merchantable and sawlog models, indicating both a small over prediction and under prediction respectively. The refit model shows that, based on a larger PSP sample size, more trees in softwood plantations are achieving merchantability and at smaller diameters compared to the original NSGNY model function. The same result was found for the ratio of trees that have sawlog dimensions relative to merchantable trees.

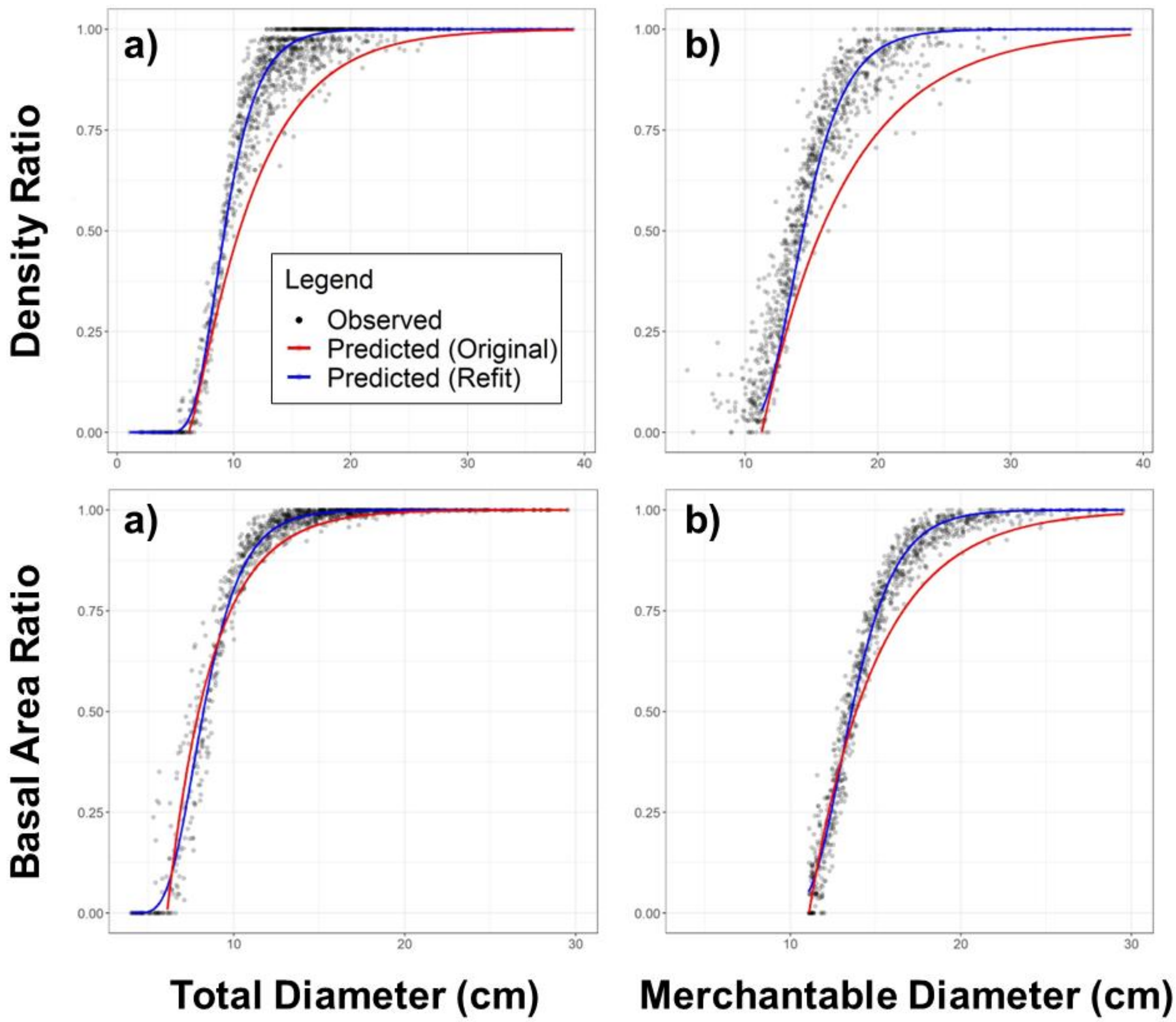


Fig. 9. Merchantability conversion ratio functions for a) merchantable and b) sawlog total density and total basal area.

3.5. Five-Year Diameter Increment

Model dTD had a RMSE of 0.6 cm and MB of 0.004 cm. In general, the updated function tended to predict lower diameter growth than the original softwood plantation function derived exclusively from red pine, while having greater diameter growth than the commercial thinning function from McGrath (2007), especially at higher stand-level basal area (Fig. 10). Diameter growth was the lowest in black spruce and highest in the other species category (Fig. 11). The dTD function included two species terms:

$$dTD = (\beta_0 sim + \beta_1)\, e^{(\beta_2 TBA)} + \beta_3 BS + \beta_4 WS$$

where dTD is five-year diameter increment (cm), sim is site index (m), TBA is total basal area (m^2/ha), BS is black spruce, and WS is white spruce.

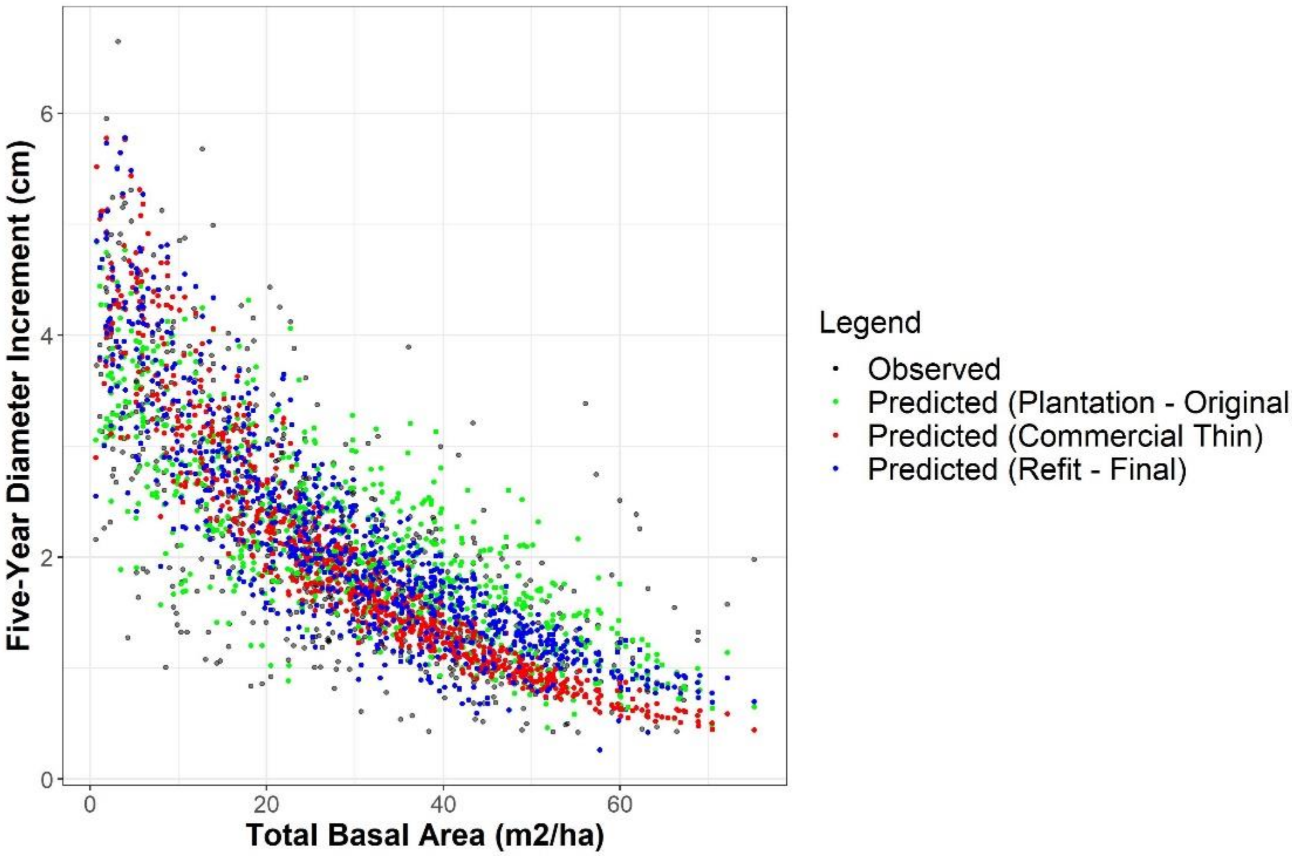


Fig. 10. Observed and predicted five-year diameter growth increment values, showing the original softwood plantation function, commercial thinning function for softwood stands, and refit commercial thinning function applied to softwood plantations.

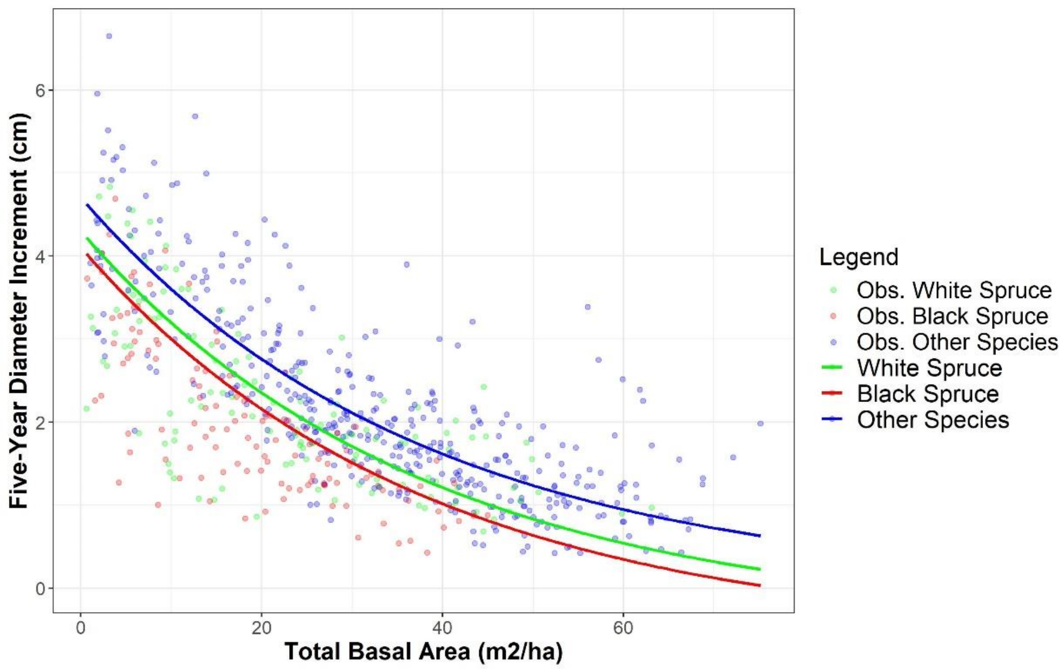


Fig. 11. Observed and predicted five-year diameter growth increment values with the refit function showing separate results for white spruce, black spruce, and other species (i.e., Norway spruce, red spruce, red pine).

3.6. Product Volume Estimates

The three new product volume ratio functions provided capacity to predict stand-level volumes of pulpwood, studwood, and sawlogs. The first function predicted the ratio of pulpwood and waste combined to merchantable volume, which had an RMSE of 0.07 and MB of 0.0007 (Fig. 12). The second function predicted the ratio of just pulpwood to merchantable volume, which had an RMSE of 0.07 and MB of -0.0008 (Fig. 13). The third function predicted the ratio of sawlog volume to sawlog and studwood combined, which had an RMSE of 0.08 and MB of -0.002 (Fig. 14).

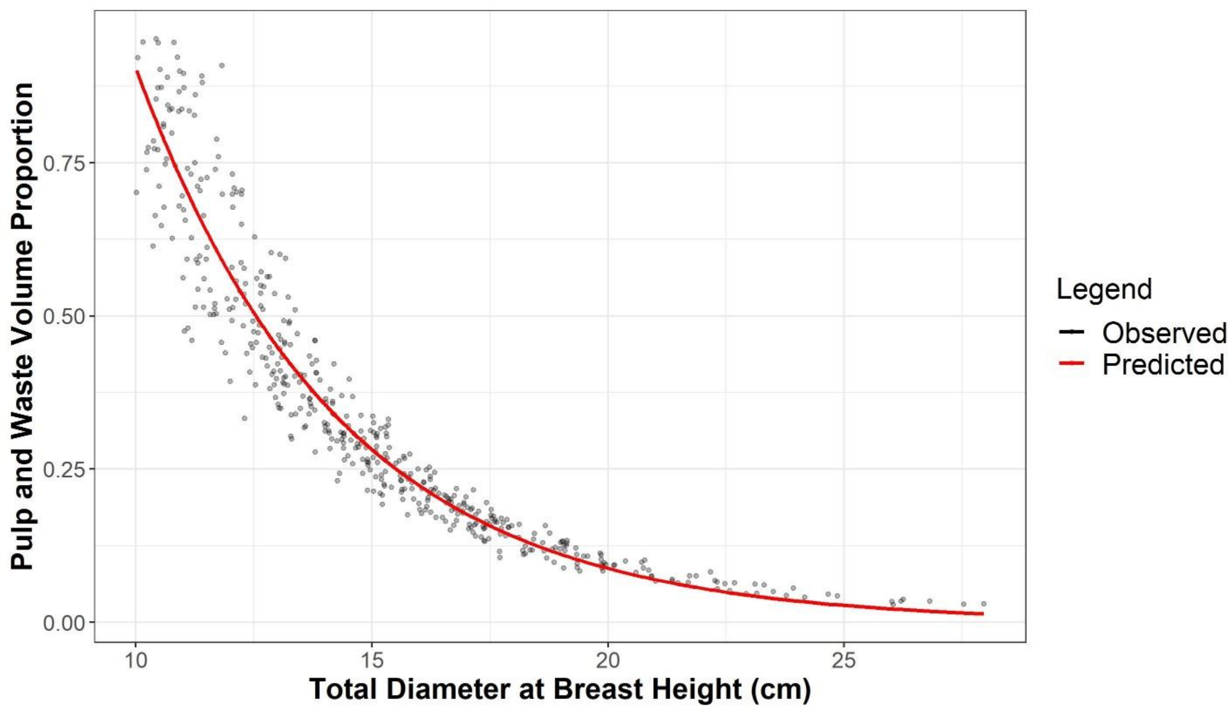


Fig. 12. Observed and predicted ratios of pulpwood and waste volume combined to merchantable volume for red spruce, white spruce, and Norway spruce.

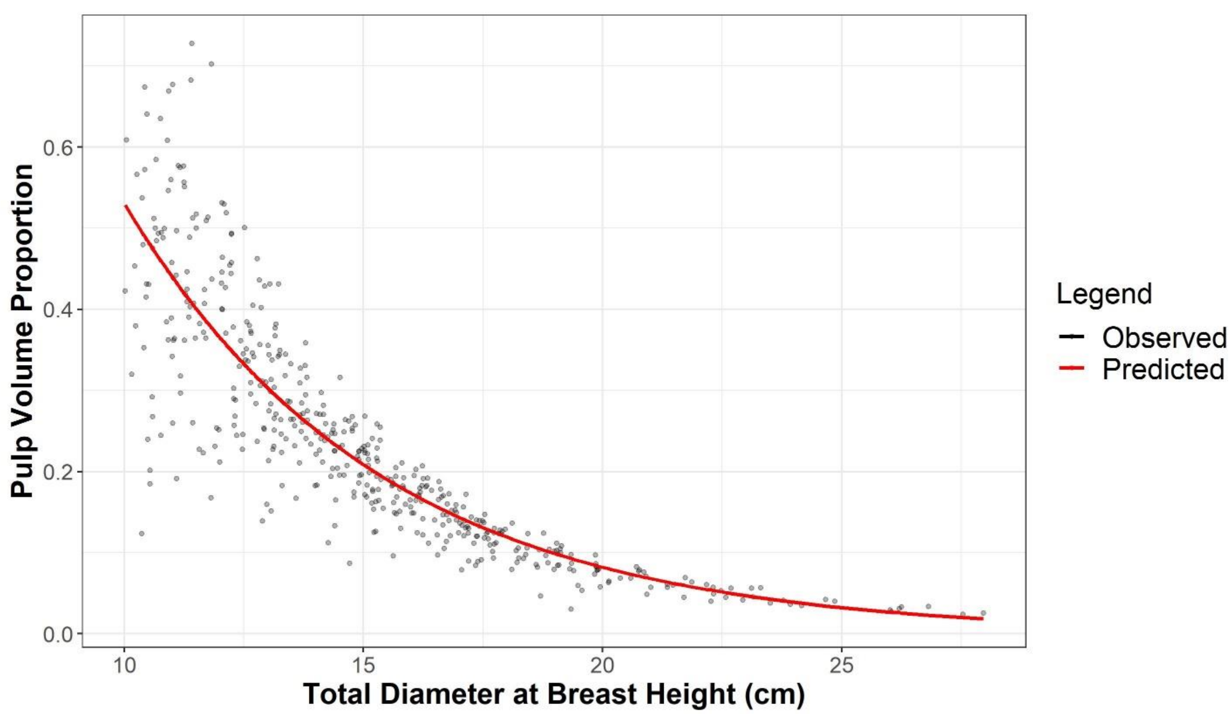


Fig. 13. Observed and predicted ratios of pulpwood volume to merchantable volume for red spruce, white spruce, and Norway spruce.

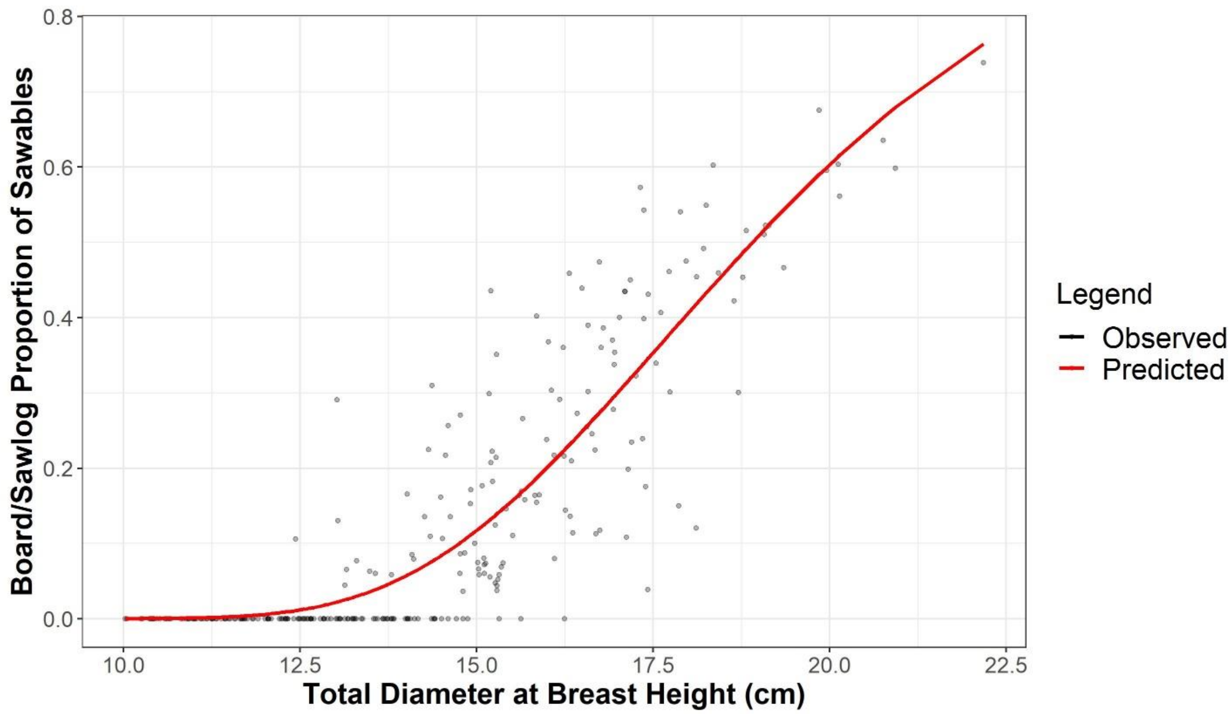


Fig. 14. Observed and predicted ratios of sawlog volume and sawlog and studwood volume combined to merchantable volume for red spruce, white spruce, and Norway spruce.

3.7. Post-Establishment Stocking Adjustment

Mortality related post-establishment stocking adjustment results are presented in Table 7. White spruce then black spruce showed higher mortality over time after plantation establishment, with lower mortality among red spruce, Norway spruce, and red pine. All functions had comparable fit, with all species except Norway spruce tending towards small under predictions.

Table 7. Post-establishment stocking adjustments for major softwood plantation species in Nova Scotia, where SAGE is stump age (yr).

Species	Linear Function	MB	RMSE
Red spruce	-0.0646397 + 0.0026711·SAGE	0195	0.133
White spruce	-0.1661021 + 0.0078457·SAGE	0238	0.159
Black spruce	-0.1554690 + 0.0097490·SAGE	0048	0.130
Norway spruce	-0.0145250 + 0.0039279·SAGE	-0396	0.153
Red pine	0.0516673 - 0.0001710·SAGE	0062	0.115

3.8. Model Comparison and Validation

Stand-level total and merchantable variables in the updated 2022 NSGNY model all showed improved model fit compared to the original 2008 model, with the sole exception of merchantable density (Table 8). Total stand variables also tended to show lower error values compared to merchantable variables. Total diameter increment is a core model function that received considerable attention in the analysis stage and had species terms added (i.e., black and white spruce).

Consequently, we also completed a model comparison and validation for species-level total diameter estimates for target species and species with an abundant sample size for softwood plantations (Table 9). White spruce, black spruce, Norway spruce, and red pine showed improvement in updated model performance against observed data compared to the original model, with red spruce showing a small decrease in performance and white pine showing a larger one.

Final 2022 model output for merchantable volume and merchantable basal area was visualized and compared against observed data from one of the larger softwood plantation research trials that contained all four spruces species as a reference (Appendix B). The comparison showed close approximation to the observed data from the research trial, with merchantable basal area beginning to over predict at older stump ages for red spruce and white spruce and under predict for black spruce and Norway spruce. The comparison shows considerable improvement over the original 2008 NSGNY model output.

Table 8. Model validation based on mean bias (MB) and root mean square error (RMSE) of output from the 2008 and 2022 NSGNY model for softwood plantations against all observed sample data (N).

	MB		RMSE		RMSE (%)	
Variable	**2008**	**2022**	**2008**	**2022**	**2008**	**2022**
TH (m)	-0.84	-0.47	1.18	1.11	13.13	12.35
TD (cm)	0.69	0.03	1.58	1.11	11.49	8.07
TF (stems/ha)	59.95	-28.81	319.76	263.28	14.16	11.66
TBA (m^2/ha)	4.16	0.20	7.91	5.20	28.98	19.05
TV (m^3/ha)	11.62	-3.46	40.63	35.54	27.76	24.28
TMAI (m^3/ha/yr)	0.25	-0.16	1.25	1.04	25.69	21.38
MH (m)	-0.87	-0.56	1.56	1.39	15.71	14.00
MD (cm)	1.28	0.74	2.77	2.33	19.25	16.19
MF (stems/ha)	-141.59	-248.91	383.45	425.86	24.20	26.88
MBA (m^2/ha)	3.84	-0.20	8.04	5.59	28.58	19.87
MV (m^3/ha)	16.07	1.22	40.26	34.56	28.91	24.82
MMAI (m^3/ha/yr)	0.50	0.08	1.25	1.02	28.47	23.23

Table 9. Model validation of total diameter by species using mean bias (MB) and root mean square error (RMSE) of output from the 2008 and 2022 NSGNY model for softwood plantations of target species against all observed plot data.

	MB (cm)		RMSE		RMSE (%)		
Species	**2008**	**2022**	**2008**	**2022**	**2008**	**2022**	**N**
Red spruce	-0.17	-0.23	0.78	0.87	5.67	6.35	109
White spruce	0.65	-0.16	1.32	1.01	10.15	7.76	173
Black spruce	1.30	0.10	1.82	0.92	17.69	8.97	318
Norway spruce	0.03	0.16	1.38	1.35	8.63	8.46	187
White pine	-0.49	-1.63	0.94	1.83	4.75	9.28	24
Red pine	0.56	-0.09	1.52	1.03	9.97	6.77	234

Recall that the model validation results shown in Table 9 differ from the results of individually fitted NSGNY functions shown in Section 3.1 to 3.7. The above are comparisons of full NSGNY model output between the 2008 and 2022 versions, so depending on the variable being validated there will be different levels of error propagation from the individual functions combined. For example, total height is one of the first variables calculated by the model and relies on only two other predictive functions (i.e., dominant height and years to breast height) and has a comparatively low level of error (Fig. 2). Conversely, merchantable volume relies on the combination of most of model functions, as well as the Li et al. (2012) taper equations, and has a comparatively higher level of error.

5. Discussion

5.1. General Reduction in Stand-Level Basal Area and Volume

The effect of all analyses and updated functions on NSGNY output is a general reduction in stand-level basal area and volume predictions. On their own, updated average height function and higher merchantable/sawlog ratios resulting from the analysis suggest gains in overall basal area and volume. However, the updated model is primarily driven by the diameter increment function, which when combined with all other functions resulted in lower net predictions of basal area and volume than the original NSGNY model. However, total and merchantable volumes do eventually catch up to original model output and start to slightly exceed it at a stump age between 50 and 60 years. Therefore, depending on rotation length (e.g., 35 years for Norway spruce versus 50 years for red spruce), merchantable volume predictions from the updated model could differ by as much as 35 m^3/ha, or by as little as roughly 3 m^3/ha above original model output.

5.2. Increase in Stand Density

In contrast to trends in basal area and volume prediction, there is an increase in predicted stand density in the updated versus original model (e.g., several hundred more stems per hectare for a 50-yr rotation red spruce plantation). This is in contrast to the simple comparison of just the density function results, where the updated function predicted a lower density at a given diameter. These results are due to slower diameter growth. Because stand density (stems/ha) is a function of total diameter, and maximum density for a given diameter occurs at a later stump age in the updated model, there is a greater density of merchantable stems for almost the entire model run in updated model results, with sawlog density beginning to exceed original model results at a stump age between 40 and 50 years.

5.3. Limitations and Uncertainties

There are some limitations and sources of uncertainty in this study. First, because of poor model fit or lack of discernable trends, some functions will need to be re-analyzed and potentially updated with further analyses of PSP data for both natural and managed stands (e.g., YTBH). Additionally, the only statistical methods used in this study were least squares linear and non-linear regression. While analyses showed improvements over the original model and reasonable levels of fit, future work might employ and compare alternative methods to further improve predictive capacity, such as mixed effects models (Weiskittel et al., 2016) or machine-learning techniques like random forests (Hamidi et al., 2021). Lastly, while this study was initiated primarily to support new analysis work for softwood HPF plantations, and therefore even-aged, single-species management, there is a need for future growth and yield research and model applications that can better handle uneven-aged silvicultural treatments and simultaneous growth of multiple cohorts and species related management recommendations in Department's Silvicultural Guide for the Ecological Matrix (McGrath et al., 2021).

The Department is engaged in research on growth and yield modelling for uneven-aged management using individual tree (a.k.a., tree list) growth and yield models like the USDA Forest Service's Forest Vegetation Simulator (FVS; Crookston & Dixon, 2005), which can be calibrated to Nova Scotia forest conditions using research and inventory PSP data. An interim area of future research could also be predicting the height and diameter distributions of stands, which was beyond the scope of this current update. Importantly, the NSGNY model was developed entirely using research data from silvicultural trials and natural stands in Nova Scotia. New growth and yield modelling research on silvicultural prescriptions that are aligned with ecological forestry (e.g., irregular shelterwoods) must be accompanied by new silvicultural trials and the generation of necessary data to advance calibrated and validated growth and yield modelling in the province. There is existing research from nearby jurisdictions with comparable forest types (e.g., Raymound & Bédard, 2017) that can inform this on-going work, but establishment of new silvicultural trials in the province will be required as part of a rigorous growth and yield program. Additionally, the model updates in this report do not include any growth gains from tree improvement, which can be substantial. The Department has an active tree improvement program, and future model updates should integrate tree improvement effects on predicted growth and yield for sites under HPF management.

5.4. Climate-Sensitive Growth and Yield Models

Another important area for future research will be integration of climate change impacts into growth and yield modelling. Since empirical models like NSGNY rely exclusively on historical data collected during a comparatively stable climate, their use in projecting future forest productivity and wood supply can be invalidated by extreme climatic change (Searls et al., 2021). The impacts of climate change on Nova Scotia forests will likely include changes in forest productivity (with potential increases and decreases), altered successional pathways and species compositions, and more frequent and severe natural disturbances (Steenberg et al., 2013; Taylor et al., 2017). Future growth and yield model advancement will need to integrate climatic modifiers under different anticipated climate change scenarios.

5.5. Conclusion

There is an ever-increasing need for evidence-based planning tools based on local, applied research to inform planning processes and policy development in Nova Scotia's forest sector. The Province has a long history of research and data collection to meet this demand and must continue to do so as the forest sector evolves with the implementation of ecological forestry and Triad zoning in a changing climate.

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Appendix A – Complete List of Current Model Functions and Coefficients

Table A1. Updated coefficient values for the 2022 NSGNY model functions assessed in this study along with all other NSGNY functions. While this study focused on plantations, the table below includes functions for all natural and managed stands as a reference.

Function Code	Coefficients β_0	β_1	β_2	β_3	β_4	β_5
YTBH	44.07021	-1.63556	0.0222	-0.000100987		
DH	-0.019070142	-3.063581805	-0.228589318	-0.019070142	3.063581805	-0.228589318
AH (natural)	-1.466091	0.861959	0.00108537			
AH (managed)	-0.53391803	0.94291819	0.03660371	0.03998594		
TD (natural)[1]	$0.286385 + 0.134989(AH-4.5) - 1.37156\cdot10^{-3}(AH-4.5)^2 + 5.46409\cdot10^{-5}(AH-4.5)^3 - 8.20707\cdot10^{-7}(AH-4.5)^4 + 7.24082\cdot10^{-9}(AH-4.5)^5 - 3.72075\cdot10^{-11}(AH-4.5)^6 + 1.02997\cdot10^{-13}(AH-4.5)^7 - 1.18654\cdot10^{-16}(AH-4.5)^8$					
TD (managed)	No coefficients					
TF (natural)	5.686293	- 1.893102				
TF (managed)	5.525482	-1.721064				
dTD	0.18480737	1.01000939	-0.02668517	-0.60056730	-0.40517823	
TBA	No coefficients					
MHrat[1]	$-15.277829 + 1.4316388\cdot AH - 0.53584452\cdot AH\cdot MR + 185.70383\cdot MR^4 + 4.6086744\cdot10^{-8}\cdot (AH/MR)^3 - 134.91889\cdot MR^5 - 68.753896\cdot MR^2 + 31.713797\cdot MR^{0.5} + 0.030025568\cdot AH/MR$					
MDrat(natural)	3.0746534	0.30615578	0.040103103			
MFrat (managed)	-0.5137051	77.3944949				
MBArat (natural)	-0.82932659	0.71839461	-0.10184704	$5.4868242\cdot10^{-3}$	$-6.6210904\cdot10^{-6}$	
MBArat (managed)	-0.6459655	140.4437701				
BHrat	9.3709292	$4.6206221\cdot10^{-2}$	$-7.4921477\cdot10^{-4}$	$4.0028957\cdot10^{-6}$	-2.8348874	3.1858712
BDrat[1](natural)	$-8.5204673 + 7.0903534(1/MD) - 1.2249943\cdot10^{-20}(MD^{10}) + 7.6542127(MD^{0.1}) - 0.2563372(MD^{0.5}) - 493.03614(1/MD^6) + 55803.496(1/MD^{10})$					
BFrat (managed)	-0.4576779	497.0174235				
BBArat (natural)[1]	$1.0794209 + 21281.691(1/MD^6) - 8.8421416\cdot10^{-13}(MD^{10}) - 6064.0629(1/MD^5) + 1.1016046\cdot10^{-11}(MD^9) - 5.0860475\cdot10^{-4}(MD^2) - 394808.99(1/MD^{10})$					
BBArat (managed)	-0.5761575	1745.1488099				
AI-AH	-3.497963	1.063977	0.075			
AI-TD-CT	0.25	0.00437				
AI-TD-PCT	0.218	0.0597	0.5			
PLP+WST	9.3232351	-0.2332636				
PLP	3.4332667	-0.1867087				
LOG	-0.2878324	159.6134991				

[1] Given the length of this function, the equation and coefficient values are all included here.

Functions:
$YTBH = \beta_0 + \beta_1 SI + \beta_2 SI^2 + \beta_3 SI^3$ Where YTBH is years to breast height (yr) and SI is site index at breast height age 50 (m)
$DH = 4.5 + (SI – 4.5) * (1 – e(\beta_0 50))^\wedge(\beta_1 SI^\wedge\beta_2) * (1 – e(\beta_3 BH))^\wedge(\beta_4 SI^\wedge\beta_5)$ Where DH is dominant height (ft), SI is site index at breast height age 50 (ft), and BH is breast height age
AH (natural) = $\beta_0 + \beta_1 DH + \beta_2 DH^2$ Where AH is average height (ft) for natural stands and DH is dominant height (ft)
AH (managed) = $\beta_0 + \beta_1 DH + \beta_2 RP + \beta_3 DHRP$ Where AH is average height (m) for managed stands, DH is dominant height (m), RP is the red pine species term (0/1), and DHRP is a red pine-dominant height interaction term
TD (natural)$_1$ – Where TD is total quadratic mean diameter (in) and AH is average height (ft)
TD (managed) = pTD + dTD Where pTD is total quadratic mean diameter (cm) from the previous period and dTD is the five-year diameter increment (cm)
TF (natural) = $10^{(\beta 0 + \beta 1 \log(TD))}$ Where TF is total density (stems/ha) and TD is total quadratic mean diameter (cm)
TF (managed) = $10^{(\beta 0 + \beta 1 \log(TD))}$ Where TF is total density (stems/ha) and TD is total quadratic mean diameter (cm)
$dTD = (\beta_0 SI + \beta_1)\ e^{(\beta_2 TBA)} + \beta_3 BS + \beta_1 WS$ Where dTD is five-year diameter increment (cm), SI is site index (m), TBA is total basal area (m^2/ha), BS is the black spruce species term (0/1), and WS is the white spruce species term (0/1)
$TBA = TD^2/40{,}000 \cdot \pi \cdot TF$ Where TD is total quadratic mean diameter (in) and TF is total density (stems/ac)
MHrat$_1$ – Where MHrat is the ratio of merchantable average height (ft) to total average height (ft), AH is total average height (ft), and MR is merchantable basal area (m^2/ha) divided by total basal area (m^2/ha)
$MDrat = \beta_0 + \beta_1 TD + \beta_1 TD^2$ Where MDrat is the ratio of merchantable diameter (cm) to total diameter (cm)
MFrat (natural) = $MBA / (\beta_0 MD^2)$ Where MFrat is the ratio of merchantable density (stems/acre) to total density (stems/acre), MBA is merchantable basal area (m^2/ha), and MD is merchantable quadratic mean diameter (cm)
MFrat (managed) = $(1 – e^{(\beta 0 TD)})^{\beta_1}$ Where MFrat is the ratio of merchantable density (stems/ha) to total density (stems/ha) and TD is total mean quadratic diameter (cm)
MBArat (natural) = $\beta_0 + \beta_1 TD + \beta_2 TD^2 + \beta_3 TD^3 + \beta_4 TD^5$ Where MBArat is the ratio of merchantable basal area (m^2/ha) to total basal area (m^2/ha) and TD is total mean quadratic diameter (cm)
MBArat (managed) = $(1 – e^{(\beta 0 TD)})^{\beta_1}$ Where MBArat is the ratio of merchantable basal area (m^2/ha) to total basal area (m^2/ha) and TD is total mean quadratic diameter (cm)
$BHrat = \beta_0 + \beta_1 MH^2 + \beta_2 MH^3 + \beta_3 MH^4 + \beta_4 BR + BR^5$ Where BHrat is the ratio of sawlog average height (ft) to merchantable average height (ft), MH is merchantable average height (ft), and BR is sawlog basal area (m^2/ha) divided by merchantable basal area (m^2/ha)
BDrat$_1$ – Where BDrat is the ratio of sawlog diameter (cm) to merchantable diameter (cm) and MD is merchantable mean quadratic diameter (cm)

BFrat (natural)[1] = BBA / ($\beta_0 BD^2$) Where BFrat is the ratio of sawlog density (stems/acre) to merchantable density (stems/acre), BBA is sawlog basal area (m^2/ha), and BD is sawlog quadratic mean diameter (cm)
BFrat (managed) = $(1 - e^{(\beta_0 MD)})^{\beta_1}$ Where BFrat is the ratio of sawlog density (stems/ha) to merchantable density (stems/ha) and MD is merchantable mean quadratic diameter (cm)
BBArat (natural)[1] – Where BBArat is the ratio of sawlog basal area (m^2/ha) to merchantable basal area (m^2/ha) and MD is merchantable mean quadratic diameter (cm)
BBArat (managed) = $(1 - e^{(\beta_0 TD)})^{\beta_1}$ Where BBArat is the ratio of sawlog basal area (m^2/ha) to merchantable basal area (m^2/ha) and MD is merchantable mean quadratic diameter (cm)
AI-AH = $\beta_0 + \beta_1 AH + \beta_2 BArem$ Where AI-AH is the artificial increase in total average height from treatment (ft), AH is total average height (ft), and BA rem is percent basal area removal
AI-TD-CT = $\beta_0 + \beta_1 TD \cdot BArem$ Where AI-TD-CT is the artificial increase in total diameter resulting from commercial thinning, TD is total quadratic mean diameter (cm), and BArem is percent basal area removal
AI-TD-PCT = $(\beta_0 + \beta_1 TD \cdot SP)\beta_2$ Where AI-TD-PCT is the artificial increase in total diameter resulting from pre-commercial thinning, TD is total quadratic mean diameter (cm), and SP is treatment spacing (m)
PLP+WST = $\beta_0 e^{(\beta_1 TD)}$ Where PLP+WST is the ratio of pulpwood and waste volume combined (m^3/ha) to merchantable volume (m^3/ha) and TD is total mean quadratic diameter (cm)
PLP = $\beta_0 e^{(\beta_1 TD)}$ Where PLP is the ratio of pulpwood volume combined (m^3/ha) to merchantable volume (m^3/ha) and TD is total mean quadratic diameter (cm)
LOG = $(1 - e^{(\beta_0 TD)})^{\beta_1}$ Where LOG is the ratio of sawlog volume (m^3/ha) to studwood and sawlog volume combined (m^3/ha) and TD is total mean quadratic diameter (cm)

[1] Given the length of this function, the equation and coefficient values are all included here.

Table A2. Post-establishment stocking adjustments for major softwood plantation species in Nova Scotia, where SAGE is stump age (yr).

Species	Linear Function	MB	RMSE
Red spruce	-0.0646397 + 0.0026711·SAGE	0195	0.133
White spruce	-0.1661021 + 0.0078457·SAGE	0238	0.159
Black spruce	-0.1554690 + 0.0097490·SAGE	0048	0.130
Norway spruce	-0.0145250 + 0.0039279·SAGE	-0396	0.153
Red pine	0.0516673 - 0.0001710·SAGE	0062	0.115

Appendix B – Perch Lake Softwood Plantation Research Trial

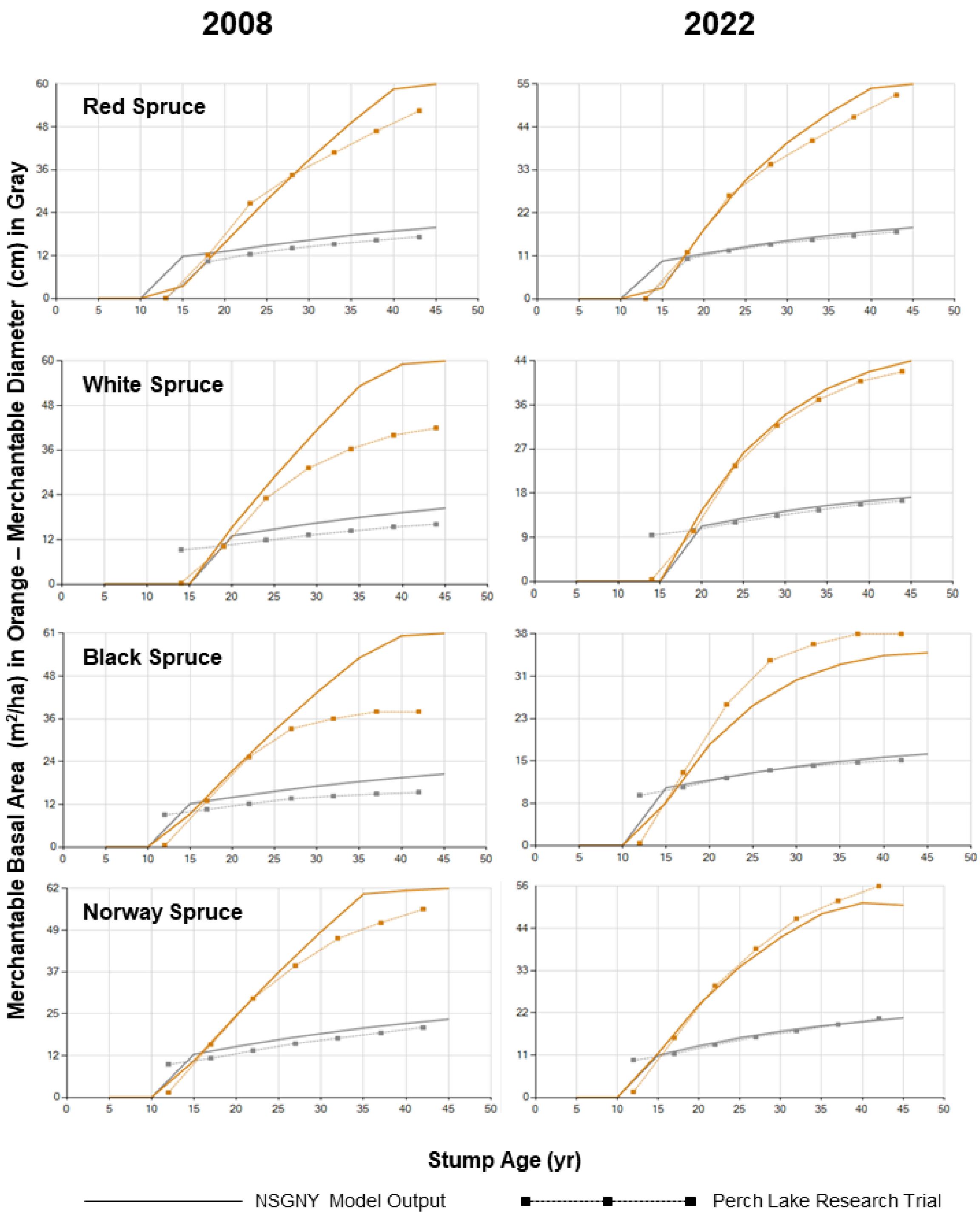

Fig. B1. NSGNY model output for total volume and total basal area from both the original 2008 model and 2022 updated model compared against the Perch Lake research trial that included all four spruce species.

Appendix C – Original Model Functions and Coefficients

Table C1. Original coefficient values for the 2008 NSGNY model functions assessed in this study, with the exception of the dTD function that was derived from McGrath, 2007. Note that all site index values are calculated using breast height age, not stump age. All units in the original NSGNY were imperial.

Function Code	Coefficients β_0	β_1	β_2	β_3
YTBH	44.07021	-1.63556	0.0222	-0.000100987
AH	-1.466091	0.861959	0.00108537	
TF	4.62277	-1.87401		
dTD	0.195	0.854	-0.033	
MFrat	-0.505737453	1.272646184		
Bfrat	-0.395102665	1		
MBArat	-0.949286627	1		
BBArat	-0.63777207	1		

Functions:

$YTBH = \beta_0 + \beta_1 SI + \beta_2 SI^2 + \beta_3 SI^3$

Where YTBH is years to breast height (yr) and SI is site index at breast height age 50 (ft)

$AH = \beta_0 + \beta_1 DH + \beta_2 DH^2$

Where AH is average height (ft) and DH is dominant height (ft)

$TF = 10^{(\beta 0 + \beta 1 \log(TD))}$

Where TF is total density (stems/ac) and TD is total quadratic mean diameter (in)

$Mfrat = (1 - e^{(\beta 0(TD - 2.4)}) ^{\beta_1}$

Where Mfrat is the ratio of merchantable density (stems/ha) to total density (stems/ha) and TD is total mean quadratic diameter (in)

$Bfrat = (1 - e^{(\beta 0(MD - 4.425711477)}) ^{\beta_1}$

Where Bfrat is the ratio of sawlog density (stems/ac) to merchantable density (stems/ac) and MD is merchantable mean quadratic diameter (in)

$MBArat = (1 - e^{(\beta 0(TD - 2.4)}) ^{\beta_1}$

Where MBArat is the ratio of merchantable basal area (ft^2/ac) to total basal area (ft^2/ac) and TD is total mean quadratic diameter (in)

$BBArat = (1 - e^{(\beta 0(TD - 4.368490128)}) ^{\beta_1}$

Where BBArat is the ratio of sawlog basal area (mft^2/ac) to merchantable basal area (ft^2/ac) and MD is merchantable mean quadratic diameter (in)

$dTD = (\beta_0 SI + \beta_1)\ e^{(\beta_2 TBA)}$

Where dTD is five-year diameter increment (cm), SI is site index (m), and TBA is total basal area (m^2/ha)

The original NSGNY model function for softwood plantations predicted total diameter and not dTD. It was based on red pine increment data only from separate increment core plantation sampling and is a function of plantation spacing, site index, and breast height age:

$$TD = 0.016384 + 1.057711\ ((0.963860501\ SP + 0.063249499\ SI - 0.179264128)\ (1- e^{(-0.029174191\ BHAGE))\text{^}0.789203213}))$$

Where SP is plantation spacing (ft), SI is site index (ft), and BHAGE is breast height age (yr).